BOOK OF ABSTRACTS
MESIGA 2019

A COMPARISON OF MEAN FIELD GAMES AND THE BEST REPLY STRATEGY

Matt Barker
Imperial College London
m.barker17@imperial.ac.uk

Abstract

In the first half of this talk we discuss the Best Reply Strategy (BRS) an instantaneous response to a cost function and how it can be related to a Mean Field Game (MFG) through a short-time rescaling. The second half focuses on stationary solutions to linear-quadratic MFGs and the BRS. We describe a new proof for existence and uniqueness of such MFGs by transforming the system into a single PDE with unknown parameters. This leads to a nice comparison of the sufficient conditions for existence between the two models. We highlight some specific examples of MFGs and BRS that highlight the type of differences that can be expected between the two models. Finally, we explain the importance of these differences, the consequences for modelling mean-field strategic behaviour, and future research questions.

AN INTRODUCTION TO DIFFERENTIAL GEOMETRY AND PORT-HAMILTONIAN SYSTEMS

Christian Bauer
RWTH Aachen University
christian.bauer@nld.rwth-aachen.de

Abstract

In an attempt to unify the mathematical modeling of complex multi-domain and multi-physics systems, Paynter, Arnold, and Brockett started to develop what is now known as port-Hamiltonian systems. In the last decades, van der Shaft, Macchelli, Kotyczka, and others refined the theory using concepts from differential geometry. The first half of the tall will introduce these concepts together with some key results obtained in the differential geometry description. In the second half, we will define a port-Hamiltonian system and showcase the most important advantage of using this formalism, which is the interconnection of port-Hamiltonian systems, exemplary for a system of two conservation laws.

Keywords
Differential Geometry, Port-Hamiltonian Systems, Interconnection

COMPUTATION OF REDUCED ORDER MODELS VIA SHIFTED PROPER ORTHOGONAL DECOMPOSITION

Felix Black
TU Berlin

Abstract

Many classical model order reduction methods are formulated in a projection framework, building the reduced order model (ROM) by projecting the full order model (FOM) onto a suitable subspace. The subspace is determined during the offline stage, usually from solving a minimization problem, and the solution of the FOM is approximated by a linear combination of elements from the subspace (modes). During the online stage, the projected system (the ROM) has to be solved for the time-dependent coefficients in the approximation. If the FOM exhibits advective transport, classical methods often fail to produce low-dimensional models with a small approximation error. One strategy to remedy this problem is the shifted proper orthogonal decomposition (shifted POD), which extends the classical POD by introducing additional transformation operators associated with the modes. The transformation operators are parametrized by paths in a suitable parameter spaces, thus allowing the transformed modes to cope with the convection. In contrast to classical methods that project onto a fixed subspace, the ROM of the shifted POD is thus obtained by projecting onto a time- and/or state-dependent subspace that adapts itself to the problem. Although this approach is very flexible, it introduces additional complexity to the online stage, since in addition to the time-dependent coefficients, also the paths need to be computed from the ROM. In this talk, we present the online stage of the shifted POD, and discuss in detail how to build a ROM from which we may compute the coefficients and the paths. Additionally, we show that for a certain class of problems, first transforming the system, projecting and afterwards applying the inverse transformation is equivalent to directly projecting onto the transformed modes, which illustrates the close relation of the shifted POD to the method of freezing. This is joint work with Philipp Schulze and Benjamin Unger (both TU Berlin).

Keywords

POD, Transport Phenomena, Model Order Reduction

LOW-RANK TENSOR-TRAIN METHODS FOR ISOGEOMETRIC ANALYSIS

Alexandra Bünger
TU Chemnitz
alexandra.buenger@mathematik.tu-chemnitz.de

Abstract

Isogeometric analysis (IgA) is a popular method for the discretization of partial differential equations motivated by the use of NURBS (Non-uniform rational B-splines) for geometric representations in industry and science. In IgA the domain representation as well as the discrete solution of a PDE are described by the same global spline functions. However, the use of an exact geometric representation comes at a cost. Due to the global nature and large overlapping support of the basis functions, system matrix assembly becomes especially costly in IgA. To reduce the computing time and storage requirements low-rank tensor methods have become a promising tool. We successfully constructed a framework appying low rank tensor train calculations to IgA to efficiently solve PDE-constrained optimization problems on complex three dimensional domains without assembly of the actual system matrices. The method exploits the Kronecker product structure of the underlying spline space, reducing the three dimensional system matrices to a low-rank format only requiring univariate integration. Thus, computation time and storage requirements are reduced significantly. The developed method automatically detects the ranks for a given domain and conducts all necessary calculations in a memory efficient low rank tensor train format. We present the applicability of this framework to efficiently solve large scale PDE-constrained optimization problems as well as an extension to statistical inverse problems using the iterative AMEn block solve algorithm which preserves and exploits the low rank format of the system matrices.

STOCHASTIC OPTIMAL CONTROL OF RENEWABLE ENERGY

Renzo Caballero
King Abdullah University of Science and Technology
renzo.caballerorosas@kaust.edu.sa

Abstract

Uruguay is a pioneer in the use of renewable sources of energy and can usually satisfy its total demand from renewable sources. Control and optimization of the system is complicated by half of the installed power - wind and solar sources - being non-controllable with high uncertainty and variability. In this work, we present a novel optimization technique for efficient use of the production facilities. The dynamical system is stochastic, and we deal with its non-Markovian dynamics through a Lagrangian relaxation. Continuous-time optimal control and value function are found from the solution of a sequence of Hamilton-Jacobi-Bellman partial differential equations associated with the system. We introduce a monotone scheme to avoid spurious oscillations in the numerical solution and apply the technique to a number of examples taken from the Uruguayan grid. We use parallelization and change of variables to reduce the computational times. Finally, we study the usefulness of extra system storage capacity offered by batteries.

THE ANALYSIS OF A BOUNDARY INTEGRAL EQUATION FORMULATION OF THE N-BODY DIELECTRIC PROBLEM IN ELECTROSTATICS

Muhammad Hassan
RWTH Aachen University
hassan@mathcces.rwth-aachen.de

Abstract

We consider the problem of calculating the electrostatic interaction between a large number of dielectric spherical particles undergoing mutual polarisation. A full description of the electrostatic interaction cannot be obtained as simply the sum of pairwise Coulomb-type interactions. Consequently, E. Lindgren et al. (The Journal of Computational Physics, 2018) have proposed a numerical method based on a Galerkin discretisation of a boundary integral equation (BIE) formulation of this problem. The proposed method is general enough to treat an arbitrary number of spherical particles of any size, charge, dielectric constant embedded in any homogeneous dielectric medium. The current talk will present results on the numerical and computational analysis of this algorithm. In particular, we derive convergence rates for important quantities of interest that are independent of the number N of dielectric particles. We show additionally that under mild assumptions, the linear system arising from the Galerkin discretisation of the BIE can be solved using O(N) operations. This allows us to conclude that the Galerkin method proposed by E. Lindgren et al. achieves linear scaling accuracy, i.e., in order to obtain a fixed average error (the total error scaled by N), the computational cost of the algorithm scales as O(N) for increasing N.

MODELING CONTACT MECHANICS AT THE BONE-IMPLANT INTERFACE

Katharina Immel

RWTH Aachen University
CNRS, Laboratoire Modelisation et Simulation Multi Echelle
immel@aices.rwth-aachen.de

Abstract

Cementless endosseous implants are widely used in orthopedic and oral surgery. However, this kind of implant still exhibits failure, like debonding. Implant failure is difficult to predict since the underlying phenomena of osseointegration and bone remodeling, especially in terms of friction and adhesion, and their influence on initial and secondary stability of the implant are still poorly understood. Especially in terms of friction and adhesion at the macro-scale, there is a lack of experimental data and reliable numerical models.
This work gives a short introduction into the relevant phenomena that influence the mechanical behavior at the bone-implant interface and the challenges that surgeons and engineers face. An example on how to solve the debonding of an osseointegrated implant is shown, by means of an analytical and a computational contact model. This contact model is based on a modified Coulomb's friction law with a displacement-dependent friction coefficient. Here, a smooth state function, based on the deformation history of the interface, is used to model implant debonding during imposed displacement. For the spatial discretization of the contact surfaces, non-uniform rational B-splines (NURBS) enriched finite elements are used, which ensure a robust and efficient contact treatment. The effectiveness of this model is demonstrated by an application to the torsional debonding of 3D titanium implants with various degrees and patterns of osseointegration.

Keywords

computational contact mechanics, sticking and sliding friction, adhesion, bone, implant, finite element method, isogeometric analysis

EFFICIENT APPROXIMATION OF A TEMPORAL MULTISCALE ODE SYSTEM

Leopold Lautsch
Otto-von-Guericke-Universität Magdeburg
leopold.lautsch@ovgu.de

Abstract

We present an efficient time discretisation scheme for an ODE system which has a multiscale character in time. We then look at this scheme in a variational context and use the dual weighted residual method (DWR) to derive an a posteriori error for an error functional J of the solution of the ODE, of the form $J(u - u_h)$. Later on, this error estimator will be used for adaptively steering the multiscale scheme by finding optimal timestep sizes.

LOCAL SOLUTIONS TO RATE-INDEPENDENT SYSTEMS: APPLICATIONS TO LARGE-STRAIN DEFORMATIONS OF DAMAGEABLE SOLIDS WITH GRADIENT POLYCONVEX ENERGIES

Petr Pelech
Charles University Prague
petr.pelech@gmail.com

Abstract

Probably the most well-known example of a rate-independent phenomenon is the dry friction. When a mug is slowly pulled on a *dry* desk, its inertia being neglected, the applied force is balanced by the friction force, whose magnitude does not increase with the speed. When the applied force varies two times faster, the movement merely rescales in time appropriately; the trajectory and the energy loss due to the friction remain the same. Although this property might seem to simplify things at first sight, it rather makes the problem degenerate and leads eventually to a loss of regularity. In many cases (e.g. damage in the fracture mechanics of solids) the processes are bound to develop jumps in time. The time derivative therefore cannot be understood in the classical sense and a proper mathematical formulation of the problem becomes a challenging task. So far, a vast family of various solutions has been introduced, with the local solution being the most general one. On the one hand, this generality provides sufficient robustness and eases the analysis; however, on the other hand, the class is so broad that there may exist even a continuum of distinct local solutions to a particular problem. The definition therefore should include additional selection criteria, which choice nevertheless still remains unclear.

In my talk I will present a proof of an existence of a local solution to problems arising in large-strain fracture mechanics. As opposed to the previous approaches dealing with small deformations, here a suitable regularization of the problem is essential for maintaining the analysis. In the second part of my talk I will introduce the notion of *gradient* polyconvexity, which provides an alternative to standard regularizing techniques.

STRUCTURE-PRESERVING SPECTRAL DIVIDE AND CONQUER METHODS FOR SELF-ADJOINT EIGENVALUE PROBLEMS

Carolin Penke
Max Planck Institute for Dynamics of Complex Technical Systems
penke@mpi-magdeburg.mpg.de

Abstract

For symmetric eigenvalue problems, spectral divide and conquer methods have been studied intensely over the years. In this class of methods, a projection onto an invariant subspace of the matrix is acquired. This is typically done via an iteration that computes the so-called matrix sign function. The projector can be used to divide the original eigenvalue problem into two smaller ones, which can be "conquered" using the same method. For this approach to work, it is important that the structure (i.e. in this case the symmetry) is preserved in the division step, s.t. the same method can be applied to the two subproblems. The notion of symmetry can be generalized with respect to indefinite inner products. We consider self-adjoint matrices with respect to inner products induced by diagonal matrices with +/- 1 on the diagonal. The ideas of spectral divide and conquer can be transferred to this pseudosymmetric setting, which includes a connection to the (generalized) polar decomposition. Applications can be found in computational quantum physics.

MULTI-LEVEL (MEAN FIELD) GAMES

Anna Thünen
RWTH Aachen University
thuenen@igpm.rwth-aachen.de

Abstract

The multi-level game is a particular subset of classical game theory. These models serve as an analytical tool to study the strategic behavior of individuals in a noncooperative manner. In particular, the individuals (players) are divided into two groups, namely the leaders and the followers, according to their position in the game. Mathematically, this leads optimization problems with optimization problems as constraints.

We discuss an application motivated two level game with a large amount of players. To overcome the challenge of high dimensionality due to the number of players, we derive the mean field limit of infinitely many players. We discuss the relation between optimization and mean field limit for different settings and establish conditions for consistency.

Keywords
Multi-Level Games, Nash Equilibria, Mean Field Games, Game Theory

LIMITED MODEL REDUCTION FOR AN ARTIFICIAL FISHTAIL

Steffen Werner
Max Planck Institute for Dynamics of Complex Technical Systems
werner@mpi-magdeburg.mpg.de

Abstract

To investigate fish-like locomotion for new types of autonomous underwater vehicles, an artificial fishtail model was created. After spatial discretization, this model is described by several hundreds of thousands of second-order ordinary differential equations. Model reduction is needed to compute an approximation that can be used for real-time control. To increase the compression of the model information, the approximation should be focused on frequency or time ranges of interest and also preserving the original model structure. Therefore, we present a limited, structure-preserving version of the balanced truncation method for second-order systems.

COUPLED ADVECTION-REACTION-DIFFUSION PROCESSES ON AN EVOLVING MICROSTRUCTURE: ANALYSIS AND HOMOGENIZATION

David Wiedemann
University of Augsburg
david.wiedemann@math.uni-augsburg.de

Abstract

We consider a porous medium composed of solid matrix and pore space, which is completely saturated with a fluid. A dissolved concentration is present in the fluid, which is affected by diffusion and advection as well as reaction at the surface of the solid matrix. This reaction causes the solid matrix to grow or shrink locally. Thus, the microstructure of the porous medium changes, which affects the transport of the concentration. In order to upscale this problem, we consider an advection-reaction-diffusion problem coupled with the Stokes equation in a domain with an evolving microstructure. The homogenization of this problem is performed utilising a transformation to a periodic reference domain and the macroscopic limit problem is determined using two-scale convergence.

GRADIENT POLYCONVEXITY IN AN ELASTOPLASTIC RATE-INDEPENDENT EVOLUTION

Jiří Zeman
University of Augsburg
jiri.zeman@math.uni-augsburg.de

Abstract

Polyconvexity is a standard assumption on hyperelastic stored energy densities which, together with some growth conditions, ensures the weak lower semicontinuity of the respective energy functional. This is essential to guarantee the existence of minimisers.

In the presentation, focus will be placed on gradient polyconvexity, introduced by Benešov, Kružík and Schlömerkemper in 2018. It is an alternative property to polyconvexity, better-suited e.g. for the modelling of shape-memory alloys. An application to large-deformation elastoplasticity will be shown in the talk, using the framework of rate-independent systems, namely the so-called energetic solutions.

The presentation is based on the author's Master's thesis, written under the guidance of M. Kružík.